Lösungsvorschlag zu Grundaufgaben zur ln-Funktion

Nr		BE

4.1

$f(x) = \frac{\ln x}{x^2}$, D_f: $x > 0 \wedge x \neq 0$: $D_f = \mathbb{R}^+$, NSt.: $x = 1$

$x \to +0 \implies f(x) = \frac{\ln x}{x^2} \longrightarrow „\frac{-\infty}{+0}" \longrightarrow -\infty$; $\lim\limits_{\substack{x\to 0\\ x>0}} f(x) = -\infty \implies x = 0$ vert. A. von G_f

$x \to \infty \implies f(x) = \frac{\ln x}{x^2} \longrightarrow „\frac{+\infty}{+\infty}"$ $l'H. \longrightarrow \frac{\frac{1}{x}}{2x} = \frac{1}{2x^2} \longrightarrow +0$

$\lim\limits_{x\to\infty} f(x) = 0 \implies y = 0$ ist horizontale Asymptote von G_f

4.2

$$f'(x) = \frac{x^2 \cdot \frac{1}{x} - (\ln x)\cdot 2x}{x^4} = \frac{x - 2x\cdot\ln x}{x^4} = \frac{x\cdot(1-2\ln x)}{x^4} = \frac{1-2\ln x}{x^3}$$

$$f''(x) = \frac{x^3\cdot\left(-\frac{2}{x}\right) - (1-2\ln x)\cdot 3x^2}{x^6} = \frac{-2x^2 - (1-2\ln x)\cdot 3x^2}{x^6} = \frac{-2-(1-2\ln x)\cdot 3}{x^4} =$$

$$= \frac{-2-3+6\ln x}{x^4} = \frac{-5+6\ln x}{x^4}$$

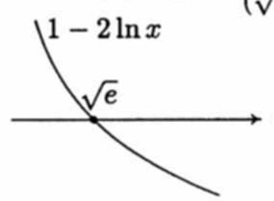
4.3

Monotonie: $f'(x) = 0$: $1 - 2\ln x = 0 \iff \ln x = \frac{1}{2} \iff x = e^{0,5} = \sqrt{e} \approx 1,65$

$$f(e^{0,5}) = \frac{\ln(e^{0,5})}{(\sqrt{e})^2} = \frac{0,5}{e} = \frac{1}{2e} \approx 0,18$$

D_f:	0 … $\sqrt{e}$	$\sqrt{e}$ … x
$1 - 2\ln x$:	+	−
x^3:	+	+
$f'(x)$:	+	−

HOP bei $\sqrt{e}$



f streng monoton zunehmend in $]0;\sqrt{e}]$ und streng monoton abnehmend in $[\sqrt{e};\infty[$ $\implies$ $H(\sqrt{e}|\frac{1}{2e})$ Hochpunkt

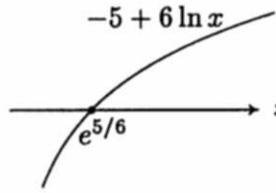
4.4

Krümmung: $f''(x) = 0$: $-5 + 6\ln x = 0 \iff 6\ln x = 5 \iff \ln x = \frac{5}{6} \iff x = e^{\frac{5}{6}} \approx 2,3$

$$f(e^{5/6}) = \frac{5/6}{\left(e^{5/6}\right)^2} = \frac{5}{6e^{5/3}} \approx 0,16$$

D_f:	0 … $e^{5/6}$	$e^{5/6}$ … x
$-5 + 6\ln x$:	−	+
x^4:	+	+
$f''(x)$:	−	+

WP bei $e^{5/6}$



G_f rechtsgekrümmt in $]0;e^{5/6}]$ und linksgekrümmt in $[e^{5/6};\infty[$ $\implies W(e^{5/6}|\frac{5}{6e^{5/3}})$ Wendepunkt

4.5